TECHNICAL ARTICLE

Two-Dimensional Vibration Analysis via Digital Holography

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Abstract

A simple modification is described to allow capture of digital holograms with two separated illuminations of a vibrating object using a standard digital holography system. The object is a small wine glass vibrating in a two-diameter mode and a three-diameter mode. Image data is resolved into axial and transverse components, and horizontal data scans are resolved into magnitude and angular direction.

Introduction

The K100 digital electronic holography system that uses the HoloFringe300K holography program has been available for a number of years, and information about it can be obtained at the website www.holofringe.com. Although often inaccurately described as a speckle interferometer, it actually makes no use of laser speckles and generates its image by numerical processing. In standard operation, it records only one component of the vectorial motion of a vibrating object. Many structures, such as cylinders, shells, and others that combine camber and twist exhibit vibrations that vary in direction across the object surface. A large class of these structures consists of blades used in jet engines, and their vibrations are essentially two-dimensional (2D) with only a small component in the root-to-tip direction. This is fortuitous, because it is much easier to construct a holographic system that can resolve vibrations in two dimensions than resolve them in three dimensions. This article describes a simple modification to the standard K100 system that allows 2D vibration measurement.

Vectorial Displacement Measurement

Vectorial displacement measurement in holographic interferometry involves making a number of recordings of the object, each of which resolves a different component of displacement. The resulting measurements are combined to generate the vectorial displacement. A single holographic interferogram measures optical phase change in the direction of what is called the **sensitivity vector**, which is defined as the propagation vector of light going from the object to the observer, $\mathbf{K}_2$, minus the propagation vector of light from the illumination optics to the object, $\mathbf{K}_1$. The argument of the fringe function, which in the case of vibration is a zero-order Bessel function of the first kind, is represented by Ω and is called as the fringe locus function. We may write

$$\Omega = (\mathbf{K}_2 - \mathbf{K}_1) \cdot \mathbf{L} \tag{1}$$

where $\mathbf{L}$ is the vectorial displacement of the object and the dot indicates a scalar product.

A comprehensive treatment of this was presented by Vest[1] in his classic book on holographic interferometry and an implementation for cylinder vibrations was performed by Tonin and Bies,[2] making use of photographic time-average holograms. Work has also been reported in this area by Sciammarella et al.[3−5] More recently, Picart et al.[6] implemented a 2D vibration solution using digital holography; however, that experimental setup is overly complex. The objective here is to present a minimal extension of an existing 1D measuring system. In the interest of simplicity, it is desirable to keep the observation of the object the

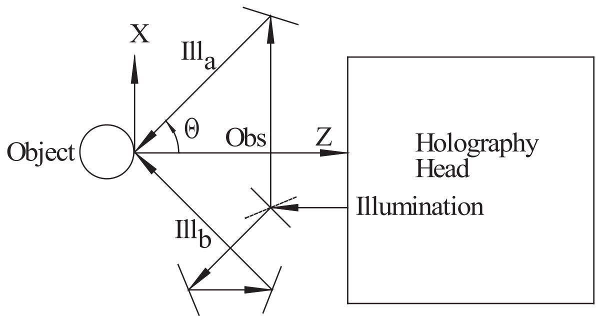

Figure 1 A diagram of a digital holography system with the illumination coming to the object from the left and right. The observation is along the *Z* axis. The first mirror the illumination strikes rotates to provide illumination beam Ill$_a$ or illumination beam Ill$_b$.

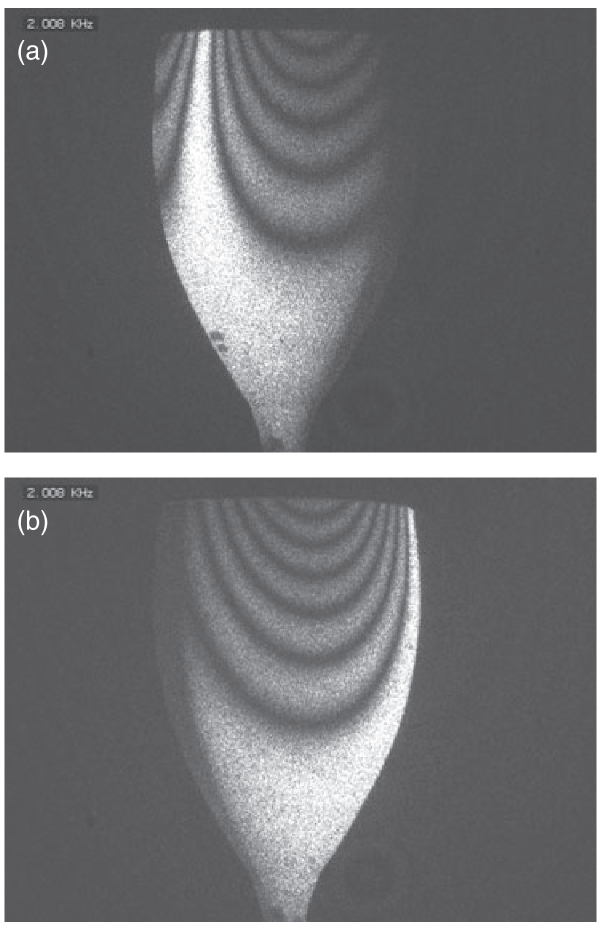

Figure 2 A two-diameter vibration of a wine glass with illumination from the left (a) and the right (b).

same for all data recordings. This eliminates the need to determine which object point in one image corresponds to the same object point in another image. The practical way to do this is to vary only the illumination vector to obtain different components of displacement. If only two dimensions are being measured, then everything can be kept in one plane, as shown in Fig. 1.

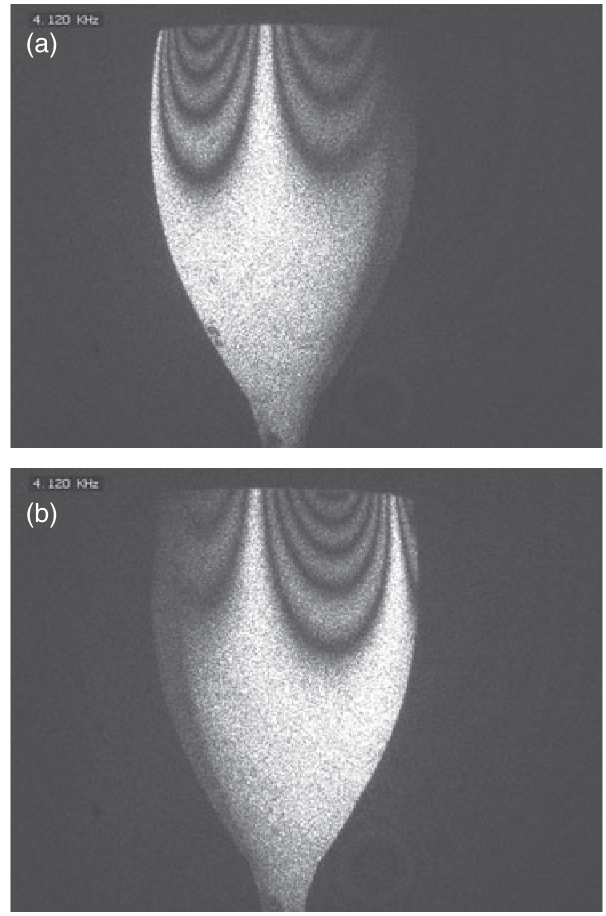

Figure 3 A three-diameter vibration of a wine glass with illumination from the left (a) and from the right (b).

The holography optical head is represented by the square with light coming out along the solid line and observation being along the *Z* axis. The mirrors have been positioned so that the optical paths are the same for each illumination, and the illuminations have been set to equal angles from the left and right of the axis along which the object is observed.

Although the HoloFringe300K program provides real-time display of J_0 fringes; it uses what is called a pseudo-phase-step process to make data recordings of vibrations. In this process, a bias vibration is applied to the reference beam at the frequency and phase of the object vibration to shift the fringe locus function in a manner analogous to phase stepping for two-beam interferometry.[7] This process requires holding the

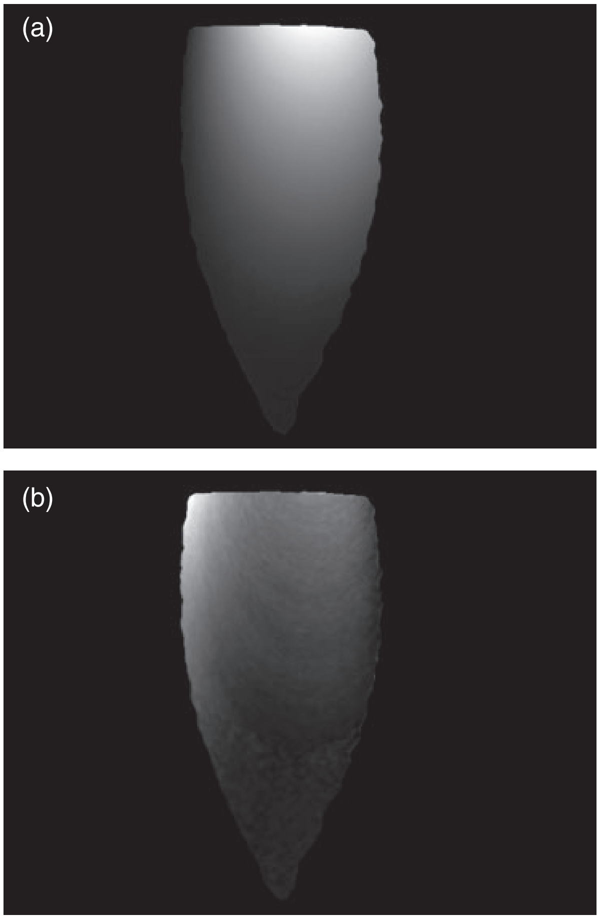

Figure 4 Axial vibration amplitude (a) and transverse vibration amplitude (b) for a two-diameter vibration mode at 2010.8 Hz.

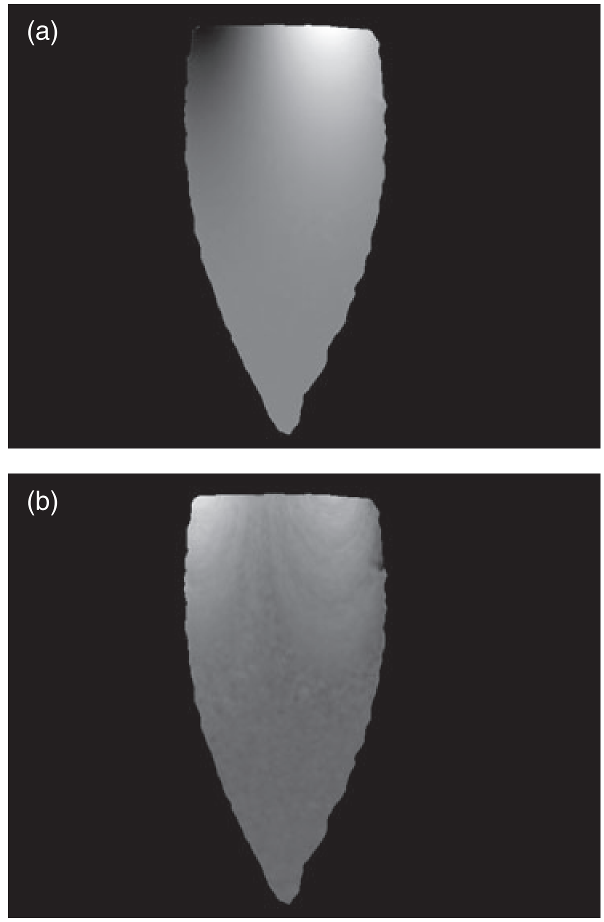

Figure 5 Axial vibration amplitude (a) and transverse vibration amplitude (b) for a three-diameter vibration mode at 4128.3 Hz.

object vibration frequency and amplitude constant over the data acquisition time, which takes in the order of 2 min. This time can easily be extended to cover two data acquisitions using illumination from one angle and then from the other. The resulting data is in the form of wrapped phase, and a collection of data sets is unwrapped in a batch process that may require a few more minutes.

In order to process the data, we must apply Eq. 1 to the geometry of Fig. 1

$$\Omega_a = (\mathbf{K}_2 - \mathbf{K}_1) \cdot \mathbf{L} = (2\pi/\lambda)\,(L_z + L_x \sin\Theta + L_z \cos\Theta)\text{, and} \tag{2}$$

$$\Omega_b = (\mathbf{K}_2 - \mathbf{K}_1) \cdot \mathbf{L} = (2\pi/\lambda)\,(L_z - L_x \sin\Theta + L_z \cos\Theta) \tag{3}$$

from which

$$\Omega_a - \Omega_b = (4\pi/\lambda)\,L_x \sin\Theta\text{, and} \tag{4}$$

$$\Omega_a + \Omega_b = (4\pi/\lambda)\,L_z\,(1 + \cos\Theta) \tag{5}$$

Experiment

The object chosen to demonstrate this was a small wine glass about 45 mm across at its top by about 110 mm high. The optical arrangement shown in Fig. 1 is set up with the illumination angles at $\pm 38^\circ$ to the observation direction. To obtain equal sensitivity to lateral and longitudinal displacement, the illumination angle would have to be 90°; however, that would only illuminate half the visible side of this object. Clearly, the preferred angle is as large as is consistent with observation of the object. In this case, 38° was the largest angle that could be accommodated by the table used. The base of the glass was set on a 3-point support, one of which was a piezoelectric driver, to which it was cemented. A vibration signal was sent to the driver from the

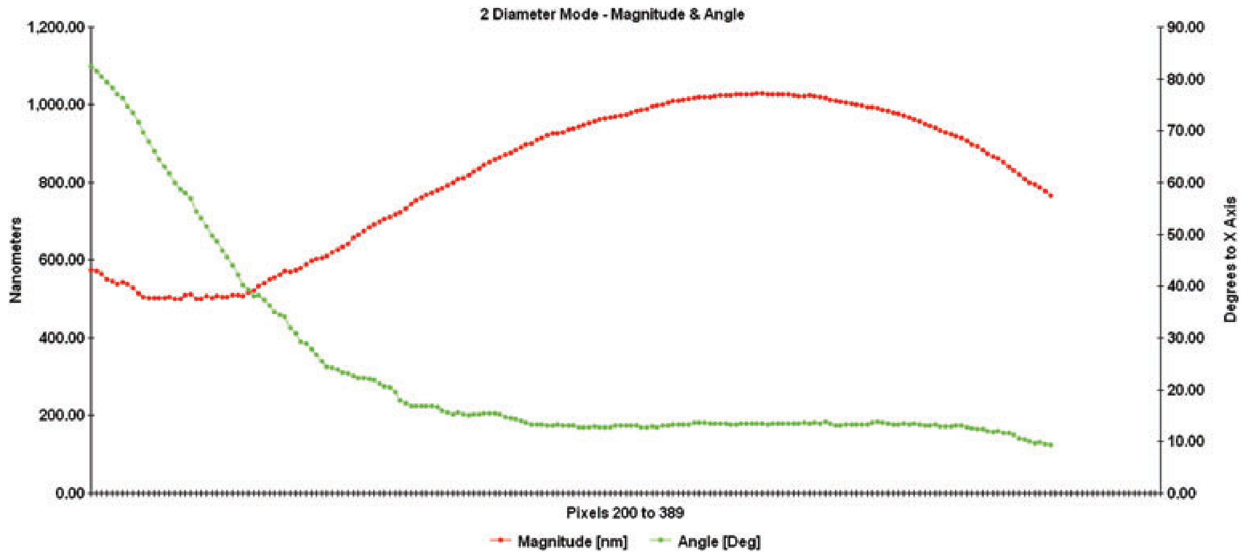

Figure 6 Vibration of the glass rim for the two-diameter vibration. The vibration is resolved into magnitude in nanometers (red) and clockwise angle from the positive *X* axis in degrees (green). Each pixel corresponds to about 0.16 mm.

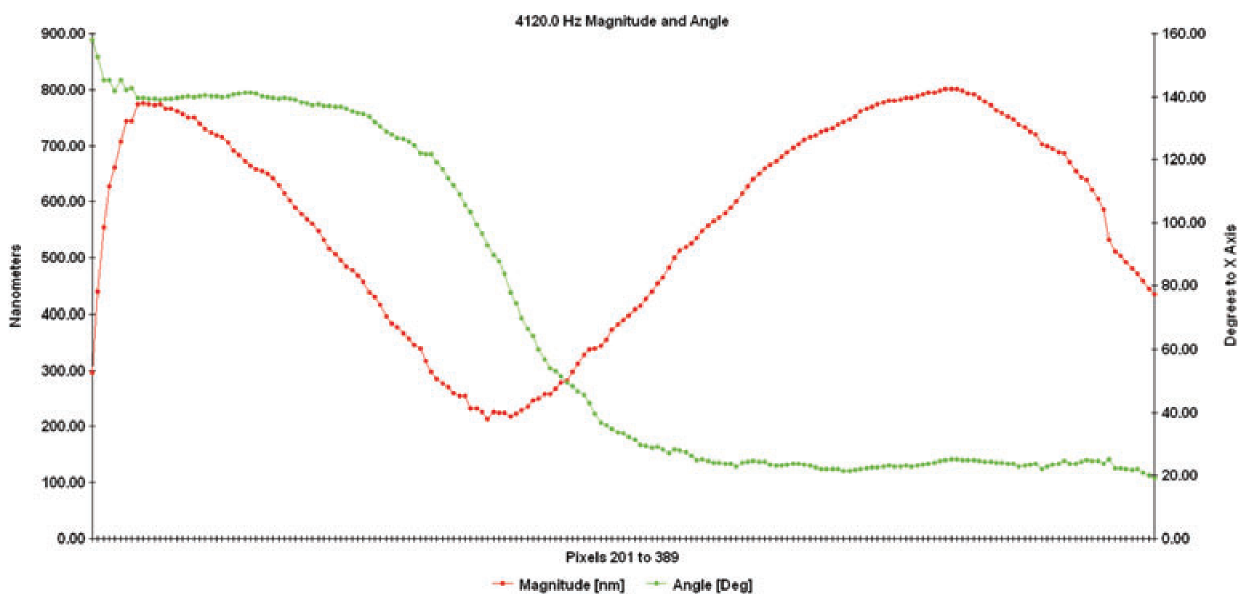

Figure 7 Vibration of the glass rim for the three-diameter vibration. The vibration is resolved into magnitude in nanometers (red) and clockwise angle from the positive *X* axis in degrees (green). Each pixel corresponds to about 0.16 mm.

signal generator controlled by the HF300K program. The glass was observed live via the hologram display and two vibration modes were noted, a two-diameter mode at 2010.8 Hz and a three-diameter mode at 4128.3 Hz. The J_0 fringe patterns for these two modes are shown in Figs. 2 and 3 with both left and right illumination.

It is clear in these images that the fringe patterns are different for the two illuminations and that the nodal lines are shifted. Data was recorded for these two modes under the left and right illuminations and unwrapped to give the fringe locus functions that are proportional to the displacements in the two sensitivity directions. These data images were processed in a mathematical spreadsheet program called DADiSP, which allowed subtraction, addition, and scaling of the data images. The data images consist of 16 bit numbers where one cycle of phase corresponds to a pixel value increase of 256. If P_a and P_b are the pixel values for the right and left illumination data recordings, we may write the equations for the axial and transverse vibratory amplitudes as:

$$L_{\text{trans}} = (P_a - P_b)\,\lambda/512 \sin(\theta)\text{, and} \qquad (6)$$

$$L_{\text{axial}} = (P_a + P_b)\,\lambda/512\ (1 + \cos(\theta)) \qquad (7)$$

The angle θ is measured as positive from the observation axis [Z] toward the right illumination. The transverse amplitude is considered positive up in Fig. 1, and the axial amplitude is considered positive toward the holography head. Images of the axial and transverse vibration amplitudes are shown in Figs. 4 and 5.

With the HoloFringe program, it is possible to read a lane of data, typically 10 pixels wide, either horizontally or vertically across an image and store the results in a file that can be read by a spreadsheet program. The program also makes it possible to scan exactly the same lane on successive images, and with this feature it was possible to acquire two rows of data for the axial and transverse vibrations, respectively, along the rim of the glass. This was done for the data images shown in Figs. 4 and 5, and a spreadsheet was used to convert these two orthogonal vibration amplitudes into magnitude and angle of the vectorial vibration. The results are displayed in Figs. 6 and 7.

Conclusions

A simple modification to the standard layout of the K100 digital electronic holography system makes it possible to record 2D vectorial vibration amplitudes for objects that are cylinders, shells, or possess twist and chamber. Two recordings are made of the vibration mode with the object illuminated from equal and opposite angles to the observation axis. The unwrapped data is processed to generate displacements in the axial (toward the observer) direction and the transverse direction. Data scans of the resulting images can be recorded for exactly the same coordinates, and these may be stored in files than can be read by a spreadsheet. Spreadsheet processing allows this data to be resolved into vibration magnitude and angle of the vectorial vibration.

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